The Fundamental Law of Mismanagement[1]

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Abstract

Grinold (1989) posits that the theoretical value added of an optimal investment strategy is approximately proportional to the product of the square root of breadth and skill. Principles derived from Grinold's "Law" are often used for designing a quantitative active investment fund. Grinold and Kahn (1995) state that: "It takes a modest amount of skill to win (the investment game) as long as that skill is deployed frequently and across a large number of stocks." They argue for frequent trading, large stock universes, and adding factors to models of forecast return. Clarke et al (2002) include a "transfer coefficient" in the formula to argue that removing constraints adds investment value, a result often used to rationalize many hedge fund strategies. We note that the formula treats asset management as a casino game that ignores estimation error and the role of inequality constraints necessary for properly defining portfolio optimality in practice. We use a simulation framework to confirm that principles commonly associated with the fundamental law are invalid and often self-defeating. Such principles have been taught in academic and professional journals for many years and likely adversely impact many hundreds of billions of dollars or more in contemporary practice.

The Grinold (1989) "Fundamental Law of Active Management," is one of the most widely referenced publications in contemporary investment theory and practice. The Grinold in-sample (mathematical) formula is based on inequality unconstrained mean-variance (MV) efficient frontier optimization translated to the residual or benchmark relative return framework. Grinold asserts that the value added of an optimized MV investment strategy is proportional to the in-sample information ratio (IR) (alpha over residual risk). He further decomposes the IR into (approximately) the product of two simple attributes of an investment strategy – square root of breadth (BR) and skill (IC).[3] The IC represents the manager's "information coefficient" or correlation of forecast and ex post return. Breadth represents the number of independent bets or factors associated with the strategy.

Based on the formula Grinold and Kahn (1995, 1999) (GK) state that: "It takes a modest amount of skill to win (the investment game) as long as that skill is deployed frequently and across a large number of stocks."[4] Their recommendations include increasing trading frequency, size of the optimization universe, and factors to models for forecasting return. There are three main assumptions that underlie their result: 1) accurate measure of IC; 2) independent sources of information, i. e. an information-rich universe that can always be mined for new independent sources of knowledge about investable securities; 3) IC the same for each component, i. e. maintainable over increases in breadth. GK use a roulette game framework to provide intuition for their results. Their principles are often used to rationalize many optimized portfolio investment strategies in current practice.

Clarke, de Silva and Thorley (2002, 2006) (CST) generalize the Grinold formula by introducing the "transfer coefficient" (TC). The TC is a scaling factor that measures how information in individual securities is transferred into managed portfolios given Grinold formula assumptions. In this in-sample framework, TC measures the presumed reduction in investment value from imposing optimization constraints. This widely influential article has been used to promote many variations of long-short and hedge fund equity investment strategies.[5]

A significant literature exists on applying the Grinold law and variations for rationalizing various active equity management strategies, particularly those associated with long-short and hedge fund portfolios. Extensions include: Buckle (2004), Qian and Hua (2004), Zhou (2008), Gorman et al (2010), Ding (2010), Huiz and Derwall (2011). Industry tutorials and perspectives include Kahn (1997), Kroll et al (2005), Utermann (2013), Darnell and Ferguson (2014). Teachings include the Chartered Financial Analyst (CFA) Institute Level 2, the Chartered Alternative Investment Analyst (CAIA) Level 1 and many conferences and graduate level courses in finance. Texts discussing the formula and applications

[3] Note that the Grinold optimization framework which is analytically derived is not to be confused with Markowitz (1952, 1959) which assumes linear (inequality and equality) constrained portfolios and requires quadratic programming techniques to compute the MV efficient frontier. In particular, the Markowitz efficient frontier is generally a concave curve even in a residual return framework while in Grinold (see e.g., GK 1995, p. 94) it is a straight line emanating from a zero residual risk and return benchmark portfolio.

[4] GK (1995, Ch. 6, p. 130), also GK (1999, Ch. 6, p. 162).

[5] One example is Kroll et al (2005). Michaud (1993) was the first to note possible limitations of the long-short active equity optimization framework.

include Focardi and Fabozzi (2004), Jacobs and Levy (2008), Diderich (2009), Anson et al (2012), Schulmerich et al (2015).

The essential wisdom of the Grinold formula – adding an independent investment significant positive source of information adds investment value – is uncontroversial. However, investment intuition supported by rigorous simulation studies demonstrate that the Grinold based GK and CST principles are invalid in practice and often defeat the effective use of investment information, even if the three demanding assumptions above are satisfied. The errors are due to ignoring estimation error in risk-return estimates and inequality constraints required for properly defining out-of-sample portfolio optimality.[6] Markowitz (1952, 1959) MV optimization, other portfolio construction procedures, and much financial theory are similarly afflicted.[7] The GK and CST principles have been taught for many years and likely adversely impact many hundreds of billions of dollars or more in contemporary asset management.

The outline of the paper is as follows. Section 1 presents the Grinold formula, the GK and CST prescriptions for active management and a critique of their casino management rationale. Section 2 discusses the four fallacies associated with GK and CST prescriptions in investment intuitive terms. Section 3 presents a framework for our simulation studies addressing the Roll (1992) critique of benchmark optimization used in Grinold. Section 4 presents the Monte Carlo simulation studies that confirm and refine our conclusion that the principles commonly associated with the fundamental law are invalid and often self-defeating. Section 5 provides resolutions for practical active optimization under estimation uncertainty. Section 6 provides a summary and conclusions.

1.0 Grinold's Fundamental Law of Active Management

Grinold (1989) demonstrates that the in-sample value added of a MV inequality unconstrained optimized residual return investment strategy relative to an index or benchmark is proportional to the information ratio (IR) (alpha over residual risk). In Grinold's "Law of Active Management" the IR of a MV optimized investment strategy is shown to be (approximately) the product of the square root of the breadth (BR) and the assumed information correlation (IC). [8] Mathematically,

$$IR \cong IC * \sqrt{BR}$$

where IR = information ratio = (alpha) / (residual or active risk)
IC = information correlation (ex ante, ex post return correlation)
BR = breadth or number of independent sources of information.

[6] The GK (1999 2nd ed.) discussion of uncertainty in IC estimation is independent of estimation error uncertainty in portfolio optimality, the subject of this paper. Esch (2015) further discusses IC estimation issues.

[7] For further discussion see Michaud (1989, 1998), Michaud and Michaud (2008a, 2008b) and Section 5.2 below.

[8] The derivation is given in GK, Ch. 6, and Technical Appendix.

The essential wisdom of the formula, that successful asset management depends on both the information level of the forecasts and the breadth associated with the estimates, is not in doubt. What is in doubt are the principles associated with the formula that the value of an optimized investment strategy necessarily increases with increasing numbers of assets in the optimization universe, number of factors in a multiple valuation framework and increased trading frequency in practical applications.

Clarke et al (2002) define the transfer coefficient (TC) as the cross-sectional correlation of risk-adjusted active weights with risk-adjusted forecasted residual returns for the N securities in the optimization universe. With additional assumptions they show that the TC can be incorporated into the Grinold formula and acts as a multiplicative scaling (a number between 0 and 1) of the IC. As constraints are added to the optimization, the TC is shown to diminish from its theoretical optimal value of one. From this point of view constraints limit the value of the information in the strategy and rationalize various long-short and unconstrained hedge fund strategies.

There are two fundamental reasons for limitations of principles derived from the Grinold law for practical asset management: 1) the formula ignores the impact of estimation error in investment information on out-of-sample investment performance; 2) the formula assumes an unconstrained MV optimization framework and ignores the necessity of including many economically meaningful inequality constraints required for defining portfolio optimality in practice. We show that GK and CST prescriptions associated with the formula are not reliable and generally not recommendable for practice.

1.1 The Casino Game Rationale

GK provide a revealing rationalization of the implications of their formula with reference to casino roulette games. In a casino the return distribution and hence the IC of plays of the roulette wheel is known, statistically significant positive, and independent. In this case the Grinold formula under the assumptions gives the (nearly) correct economic value of the play of the casino game. In contrast, the IC of the play of an investment game has estimation error, is by definition unknown, may be insignificant or even negatively related to return, and may not be independent—the IC may change if the frequency of investing or the size of the universe is changed. Increasing the number of plays of an investment game if the return distribution or IC is unreliably positive may often be undesirable.

While the GK and CST frameworks assume no constraints on the number of plays of the roulette game, there are practical limits. It is not possible to expand the number of roulette wheels infinitely. There are limits to the ability of croupiers to speed up the game and too frequent plays may reduce the “take” for each play. While adding more roulette tables to the casino may be theoretically beneficial, there are costs including limits to floor space, adding more croupiers, more advertising and other expenses. Also, there are limits to sustainable gaming interest in any community. More plays of the game in practice, may often be infeasible or even undesirable. Similar constraint limitations also exist in asset management and will be discussed further below. However, our simulation study shows that even in the absence of these types of real-world limitations, the presence of estimation error causes diminishing performance gains, and a cumulative underperformance relative to the predictions of the fundamental law.

GK principles apply to casino games where estimation error and important practical issues are ignored. But investing is not a casino game and asset managers do not know with certainty the risk and return distribution ex ante in practice.[9] Note that casino return certainty is also assumed in many other portfolio construction procedures in investment practice and much financial theory.[10] Their results follow from a theoretical framework that ignores the transcendent importance of estimation error and economically valid inequality constraints for properly defining portfolio optimality in practice. The following sections more formally explore the implications of estimation error and inequality constraints on the GK and CST proposals for asset management.

2.0 Discussion of GK and CST Prescriptions

The four principles that are associated with GK and CST are: 1) increasing size of the optimization universe; 2) invest often or frequent trading; 3) add factors to forecast model; 4) remove constraints. In this section we discuss the limitations of each prescription from an intuitive point of view.

2.1 Large Optimization Universe Fallacy

GK argue that investment value increases as the size of the optimization universe conditional that the IC is roughly equal for all securities in a given optimization universe. How realistic is this assumption?

For a small universe of securities the assumption of uniform average IC may be tenable. Small universes may be fairly homogeneous in character. However, for a large and expanding optimization universe, it seems evidently untenable to assume uniform average IC across all subsets. Any manager will naturally use the securities with the best information first. While theoretically, adding more assets may add marginally to breadth, all other things the same, it is also likely to result in less predictable securities and reducing the overall average IC level of the universe. A lower level of average IC is undesirable according to the formula and may cancel any gains made by increasing breadth by increasing size.

The issue can be framed in a more common practical setting. Consider an analyst suddenly asked to cover twice as many stocks. Given limitations of time and resources, it is highly unlikely that the analyst's average IC is the same for the expanded set of stocks. Issues of resources and time rationalize why analysts tend to specialize in areas of the market or managers in investment strategies that limit the number of securities that they cover. In practice many traditional managers limit the number of securities they include in their

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[9] The use of insider information is illegal under U.S. security laws.

[10] A game framework has long been associated with the history of probability and applications. In this case there is assumed a known or underlying stable probability distribution associated with inference. More recently the notion of probability that includes uncertainty or estimation error has been found important in many applications in business and social sciences such as finance. For an up-to-date discussion see Weisberg (2014).

active portfolio to not much more than twenty or fifty. Except for relatively small asset universes, the average IC and overall level of IR may often be a decreasing function of the number of stocks in the optimization universe, all other things the same. Grinold and Kahn seem to be aware of these limitations, for example as suggested by their statement "The fundamental law says that more breadth is better, provided the skill can be maintained." Nevertheless, this important caveat is omitted from their final summary and often ignored by practitioners who may not have accurate knowledge of their true IC, especially when coverage increases to larger universes of assets and/or factors.

2.2 Multiple Factor Model Fallacy

Large stock universe optimizations may include the S&P500, Russell 1000 or even a global stock index as benchmarks. In this case individual analysis of each stock is generally infeasible and analysts typically rely on factor valuation frameworks for forecasting alpha. For example, stock rankings or valuations may be based in part on an earnings yield factor.[11] As GK note, if earnings yield is the only factor for ranking stocks, there is only one independent source of information and the breadth equals one.

In the Grinold formula, BR is identified with the number of uncorrelated investment significant positive factors used to forecast alpha. The formula shows that the IR increases as the square root of the number of independent positive significant factors in the multiple valuation forecast model. However, in practice, asset valuation factors are often highly correlated and may often be statistically insignificant providing dubious out-of-sample forecast value.[12] Finding factors that are reasonably uncorrelated and positive investment significant relative to ex post return is no simple task.

Factors are often chosen from a small number of categories considered to be relatively uncorrelated and positively related to return such as value, momentum, quality, dividends, and discounted cash flow.[13] In experience, breadth of multiple valuation models is typically very limited and unlikely to be very much greater than five independent of the size of the optimization universe.[14,15] As in adding stocks to an optimization universe, adding factors at some point is likely to include increasingly unreliable factors that are likely to reduce, not increase, the average IC of an investment strategy.

Michaud (1990) provides a simple illustration of adding factors to a multiple valuation model. While adding investment significant factors related to return can be additive to IC, it can also be detrimental in practice. There is no free lunch. Adding factors can as easily

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[11] Some standard methods for converting rankings to a ratio scale to input to a portfolio optimizer include Farrell (1983) and references. Michaud (1998, Ch. 12) notes some common scaling errors.

[12] There is a limit to the number of independent investment significant factors even in many commercial risk models, often far less than ten.

[13] Standard methods such as principal component analysis for finding orthogonal risk factors are seldom also reliably related to return.

[14] See e.g., Michaud (1999).

[15] While principal component or factor analysis procedures for identifying orthogonal factors in a data set may be used, most studies find no more than five to ten investment significant identifiable factors that are also useful for investment practice.

reduce as well as enhance forecast value, and the number of factors that can be added while maintaining a desirable total IC is severely limited in practice.

2.3 Invest Often Fallacy

GK recommend increasing trading period frequency or "plays" of the investment game to increase the IR of a MV optimized portfolio. The Grinold formula assumes trading decision period independence and constant IC level. However, almost all investment strategies have natural limits on trading frequency.[16] For example, an asset manager trading on book or earnings to price will have significant limitations increasing trading frequency smaller than a month or quarter. Reducing the trading period below some limit will generally reduce effectiveness while increasing trading costs.

Fundamentally, trading frequency is limited by constraints on the investment process relative to investment style.[17] Deep value managers may often be reluctant to trade much more than once a year while growth stock managers may want to trade multiple times in a given year. Increased trading, to be effective, requires increasing the independence of the trading decision while maintaining the same level of skill. This will generally require increased resources, if feasible, all other things the same. The normal trading decision period should be sufficiently frequent, but not more so, in order to extract relatively independent reliable information for a given investment strategy and market conditions.

It is worth noting that the notion of normal trading period for an investment strategy does not imply strict calendar trading. Portfolio drift and market volatility relative to new optimal may require trading earlier or later than an investment strategy "normal" period. In addition a manager may need to consider trading whenever new information is available or client objectives have changed. Portfolio monitoring relative to a normal trading period including estimation error is further discussed in Michaud et al (2012).

2.4 Remove Constraints Fallacy

Markowitz's (1952 1959) MV optimization can accommodate linear equality and inequality constraints. In actual investment practice, MV optimized portfolios typically include many linear constraints. This is because MV optimized portfolios are often investment unintuitive and impractical. Constraints are often imposed to manage instability, ambiguity, poor diversification characteristics, and limit poor out-of-sample performance.[18] However, constraints added solely for marketing or cosmetic purposes may result in little, if any, investment value and may obstruct the deployment of useful information in risk-return estimates.

In general, inequality constraints are necessary for defining portfolio optimality in practice. Inequality constraints reflect the financial fact that even the largest financial institutions

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[16] Special cases may include proprietary trading desk strategies where the information level is maintained at a reasonable level and trading costs are nearly non-existent. Other cases, such as high frequency and algorithmic trading are arguably not investment strategies but very low level IC trading pattern recognition relative to highly sophisticated automated liquidity exchange intermediation.

[17] Trading costs and market volatility are additional considerations.

[18] Jobson and Korkie (1980, 1981), Michaud (1989).

have economic shorting and leveraging limitations. Recently, Markowitz (2005) demonstrates the importance of practical linear inequality constraints in defining portfolio optimality for theoretical finance and the validity of many tools of practical investment management. Long-only constraints limit liability risk, a largely unmeasured factor in most portfolio risk models and often an institutional requirement. Regulatory considerations may often mandate the use of no-shorting inequality constraints. Performance benchmarks may often mandate index related sets of constraints for controlling and monitoring investment objectives.

In an important early study, Frost and Savarino (1988) demonstrate that sign or non-negative inequality constraints may limit the impact of estimation error and consequently improve out-of-sample investment performance, contradicting CST. This is because economically valid constraints act like Bayesian priors focused on portfolio structure rather than the return estimation by enforcing rules representing legitimate information not contained in the optimization. Such restriction can mitigate estimation error in risk-return estimates implicitly by forcing the solutions towards more likely optimal portfolios.

3.0 Residual and Total Return MV Optimization

The Grinold MV active management framework reflects much investment practice. An active manager is typically hired to explicitly beat some benchmark such as the S&P 500 or Russell 1000 index while limiting tracking error or residual risk. The mathematical consequence of using a residual rather than total return framework is simply a revision of the MV optimization budget constraint to sum to zero instead of one relative to index weights, and a consequent reformulation of the variance component of the utility objective. Interestingly, contemporary commercial equity risk models are often defined for either total or residual return optimization. The notion is that the optimal MV residual return portfolio for a specified index tracking error can also be estimated as a total return MV optimization calibrated to the desired tracking error. However, any practical benefits associated with the residual return MV optimization framework may often be associated with serious investment limitations.

3.1 The Roll Critique

Roll (1992) provides a serious critique of the Grinold active return MV optimization framework. Roll shows that if the index is not total return MV efficient all the portfolios on the index relative efficient frontier are dominated in total return MV space. This means that there are always portfolios with less risk or more estimated return or both than any portfolio on the residual return efficient frontier. The presumed convenience of optimizing a portfolio relative to a given benchmark can lead to very poor investments. On the other hand, as Roll notes, if the index is MV efficient, the total and residual return efficient frontiers coincide and the residual return optimized portfolios are also total return MV efficient. In this case portfolios on the MV total return efficient frontier are also residual return MV efficient relative to some level of tracking error and maximization of IR is equivalent to max Sharpe ratio (MSR) optimization in total return space.

From the point of view of rational markets, it is hard to justify the IR optimization framework if the index is not, in some investment meaningful sense, at least approximately

MV efficient.[19] We note that a framework where the Grinold assumptions are also valid for total return MV optimization is a best case for understanding the practical investment limitations of the GK and CST prescriptions. It is also important to avoid the Roll critique as an explanation for poor performance in our simulation experiment, which is designed to rigorously demonstrate the limitations of GK and CST principles in the context of estimation error.

3.2 Merton (1987) and Benchmark MV Efficiency

In an important paper for investment practice, Merton (1987) proposes an information cost structure model of MV market equilibrium. In his study he presents relatively simple conditions under which common benchmarks in active management practice may be considered essentially total return MV efficient.

Merton's incomplete information framework posits constraints and information levels where investors act as if they do not know many firms in large capital markets.[20] In this case, the cost of information limits the market portfolio that can be efficiently considered for investment. For example, a small cap stock manager may claim specialized expertise for managing a benchmark of small cap stocks relative to a larger universe of securities but little if any for large cap. Institutional investors often consider it optimal to hire managers with specialized expertise in different segments of the market, hoping that the total will exhibit a globally enhanced level of return for given risk level.

In the Merton model, expected return reflects a discount factor for the subset of information available stocks. As Merton notes, there are many possible frameworks for justifying the notion of market equilibrium in incomplete information. These may include prudent-investing laws, regulatory constraints, and short-sale proscriptions.

Merton's framework rationalizes a variety of contexts that are consistent with actual sophisticated asset management. In the incomplete information case and in variations, the benchmark can be assumed essentially total return MV efficient. Alternatively, the contrapositive of the generalized Merton framework implies the existence of managers willfully managing money inefficiently and sophisticated investors and institutions willingly investing in such strategies, a contradiction of rational markets. As Roll notes, the notion of an index that is not, in some fundamentally meaningful sense, MV efficient raises important investment issues independent of those in this report.

Our working assumption is that the benchmark can be assumed MV efficient relative to some set of constraints and assumptions including information costs. In this case the IR characteristics of the unconstrained MV optimal portfolio in a residual return framework can be equivalently analyzed with respect to the properties of maximum Sharpe ratio

[19] One rationale noted by Roll is that estimation error is so extensive that the benchmark may not be statistically indistinguishable from MV efficiency.

[20] Note that a benchmark consistent with the Merton incomplete information framework requires economic considerations and is inconsistent with actuarial based liability driven investing (LDI) popular with many actuarial and some pension consulting firms. See Michaud (1998, Ch. 10) and associated references for defining a liability benchmark based on economic principles.

optimal portfolios in total return MV space. We will often refer to maximizing IR and MSR interchangeably.

4.0 GK and CST Simulation Proofs

While section 2 provides a number of challenges to many of the notions of GK and CST for practice, the narrative is largely based on investment intuition and practice rather than rigorous demonstration. In this section we address the limitations of the GK and CDT for practice within a rigorous simulation framework.[21]

4.1 Jobson and Korkie Simulation Studies

The Grinold MV framework assumes estimation error free unconstrained MV optimization. Jobson and Korkie (1981) (JK) provide the classic study of the effect of estimation error on the out-of-sample investment value of inequality unconstrained MV optimized portfolios. By means of a simulation study designed much like the one in this paper, they show that the additional performance gain from an unconstrained MSR optimization is more than cancelled out by the loss incurred by a realistic level of estimation error, i. e. that equal weighting substantially outperforms unconstrained Sharpe ratio maximization under a realistic amount of estimation uncertainty. Their study is performed with total-return optimization rather than benchmark-relative active-weight optimization, but their result can be extended to active weight optimization since the optimal frontiers are identical if the benchmark is efficient, and if not, the performance of the active-weight optimization can only be worse since the active-weight frontier is mathematically dominated everywhere by the total-return optimal one.[22]

In contrast to JK study's use of estimation periods as a proxy for the degree of estimation error and thus the information level of the analysis in their experiment, GK use average IC and not estimation periods to represent information level in practice. For example, an equity portfolio manager may claim an average 0.10 IC to reflect their anticipated correlation between forecasts and ex post returns for a given investment strategy. However, the two concepts are closely related; i.e., increasing estimation periods increases the IC of the simulated forecasting process.[23] It is important to note that the average IC associated with a given number of estimation periods also depends on the risk-return distribution of the case-specific optimization universe. While estimation periods may be a more reliable engine to drive estimation error in a Monte Carlo simulation process, we will use the Grinold framework IC to define ex ante information level in our simulations.[24]

4.2 Simulating Breadth while Maintaining Information Levels

[21] Note that simulation proofs used in many recent studies are superior to any back tests of investment effectiveness. A back test is always time period dependent and unreliable out-of-sample.

[22] Michaud (1998, Ch. 4) replicate the JK simulation studies based on a data set of six diversified country equity and two bond indices and found qualitatively similar results considering the different character of the historical risk-return distribution.

[23] The 60 estimation periods in the JK study represents roughly an IC of 0.45.

[24] Simulations were designed to attain a particular level of average IC by combining the target mean with some independently sampled noise. More information is available in Esch (2015)..

One of the fundamental GK precepts is the notion that simply adding securities as a way of adding breadth (BR) leads to improved MV optimized out-of-sample investment performance all other things the same. In the following we generalize the JK simulation framework to illustrate the impact of estimation error on out-of-sample investment value relative to optimization universe size, while maintaining a constant IC across all universe sizes. Although the case has been made that there are practical limits in the real world for increasing breadth while maintaining IC, we do not wish any failure in performance as breadth increases in our experiment to be attributable to a loss in IC.

Although the number of assets is not identical to breadth as specified by GK and CST, the particular construction of the simulated covariance matrix that we are using guarantees that we are adding breadth as we continue to add assets to the case, since each asset is given some idiosyncratic variance in the model, and the covariance is guaranteed to have full rank.[25] Basing an estimation process on data from a greater number of assets in this simulation framework provides new independent information since each asset has a residual variance that is partly explained by the increased cardinality of the estimation. It is never true that we are using the best assets that explain the most of the total variance of all the assets first, as would likely be the case in a real investment situation. We are still adding breadth up to the last increment in portfolio size, and the results cannot be explained as breadth leveling off as a function of portfolio cardinality.

4.3 Simulation Methodology

We calculate a truth for the purposes of simulation based on monthly data taken from the Russell 1000 index.[26] Our simulation provides good coverage of the return distribution from a recent history (2012-2013) of this index, while our methodology guarantees a limitless supply of simulated breadth at a constant IC.[27]

An important factor that can greatly reduce the average IR of an optimized portfolio is ill-conditioning of the covariance matrix. When the covariance matrix is calculated using the sample covariance of historical data, this performance-killing effect creeps in as the number of assets approaches the number of time periods in the estimation process. In practice, the covariance matrix for an equity optimization is likely to be obtained as the

[25] A number of earlier versions of the Grinold law misidentified the N in breadth as the number of stocks.

[26] We include all listed U. S. stocks in the CRSP database that had two years of continuous monthly returns from January 2012 to 2013. We excluded returns greater than 50% or less than -50%. We found 5307 stocks that met our criteria.

[27] Choosing only securities with a fixed minimum available history creates survivorship bias and paints an overly positive portrait of expected return. In order to compromise between this selection bias and a realistic return distribution including returns from the low end of the spectrum, we have limited the historical data requirement to two years. This selection criterion still errs on the side of optimism, since real-world baskets of selected stocks are likely to produce returns biased negatively relative to the predictions of the experiment. Although using only two years of history definitely introduces estimation error, in aggregate the set of estimates, with estimation error, should provide good coverage of the true return distribution of real stock returns of investable stocks. We also performed simulation experiments with other datasets with different history requirements, and reached exactly the same conclusions Although using only two years of history definitely introduces estimation error, in aggregate the set of estimates, with estimation error, should provide good coverage of the true return distribution of real stock returns of investable stocks. These supplemental experiments are not shown here due to space limitations.

output of a factor model complete with an idiosyncratic variance term for each asset. By construction these estimates for the covariance matrix will always be full rank and reasonably well-conditioned, and performance as measured by IR will not suffer because of ill-conditioning. In our experiment we do not wish to test the impact of near-singularity of the covariance matrix, so we simulate the variance parameters (not the estimates) of the entire sampling pool of assets in such a way that well-conditioning is guaranteed for portfolio sizes up to 500 in our examples. This is almost certainly optimistic with respect to practice, so our findings on the out-of-sample performance of the fundamental law's predictions represent a best-case scenario, and real world applications are likely to fare worse.

We use the direct estimates from the real two year histories to represent truth in the simulation experiment. From this "true" return distribution we simulate estimates which would correspond to the inputs in a practitioner's optimization. These estimates are designed to be consistent with their targets while including estimation error, all maintaining both a particular expected IC for the mean estimates[28] and a well-conditioned covariance estimate.[29]

Portfolios are then created from the simulated mean and variance inputs via three methods: unconstrained maximum Sharpe ratio, maximum Sharpe ratio with positivity constraints, and equal weighting. Of course numerous other methods are possible, but not presented here due to space limitations. Information ratios are then calculated for each method using the population values. These are not the in-sample information ratios that an investor would calculate using his or her own estimates; they are the true population information ratios which are calculable only within the experiment using the simulation parameters.

[28] Details of simulating with a particular IC are given in Esch (2015). The nominal ICs shown in this experiment are likely inflated from their counterparts in the real world, since ICs are typically calculated as the correlations between estimates and realized returns, rather than true return expectations. As shown in Esch (2015) this has the effect of inflating the IC by the ratio of the total standard deviation of the realized returns divided by the standard deviation of the selected expected return due to the portfolio sampling process.

[29] Procedurally, we use the Ledoit (2003, 2004a, 2004b) covariance estimator on a short simulated history for the sampled subset within each simulation cluster, which creates a stable and full-rank estimate even when the dimensionality of the matrix exceeds the sample size. We use a small sample size of only ten observations here to create some error about the true covariance matrix. We feel that it is necessary to introduce some estimation error since the model assumption of a stationary covariance matrix is likely to be false, and in spite of their popularity and marketing claims, real-world factor models come with estimation error. In our experiments we found that larger sample sizes created estimates that were for practical purposes too close to their population values. After the "model" covariance is calculated as the Ledoit estimate of the dataset, repeated subsamples are drawn that are tightly clustered around these model covariances, by sampling from a Wishart distribution with degrees of freedom safely greater than the largest sample size in the experiment, in order that the matrices used in optimization will never suffer from ill-conditioning problems. This two-step simulated estimation process may seem unnecessarily complicated, but it successfully avoids the performance-killing ill-conditioning typical of matrices drawn from the thin information available for maintaining reasonably current estimates, and provides a good simulation of the estimation error implied by the paucity of relevant data to the current time period. Thus the step that provides the estimation error is the Ledoit step, taken with only ten time periods, and multiplicity is provided in the Wishart step, which is also done each time with a different set of assets, so the tight clustering of estimates provided by this method will not matter since each estimate is for a different asset mix.

Generally the in-sample estimates are far too optimistic, and indeed, although they are not shown on these graphs, their ranges dominate the others on the graph. For reference we also calculated the ranges of theoretical true maximum Sharpe ratios using the population parameters. Of course in practice these portfolios would be unattainable since they represent flawless estimation when only flawed information is obtainable in practice.

In the simulation studies that follow we differentiate two cases that reflect investment practice: asset allocation and equity portfolio optimization strategies. Asset allocation strategies typically include five to thirty securities and rarely more than fifty. On the other hand equity portfolio optimization strategies may include hundreds or even thousands of assets in the optimization universe. In both asset allocation and equity portfolio optimization we consider IC values of 0.10 and 0.50. While active equity asset managers may often claim to have an IC level of approximately 0.10 a more optimistic IC of 0.50 may be useful to bracket our results illustrating GK principles.

4.4 Russell 1000 MV Optimization Simulation Results

Figures 1 and 2 each consist of two panels of simulation results corresponding to IC levels of 0.10 and 0.50. Figures 1A and 1B display simulation results for universe sizes up to 50 stocks representing the asset allocation case. Figures 2A and 2B display simulation results for universe sizes up to 500 stocks and represent the equity portfolio optimization case. Four sets of ranges are displayed in each panel, each showing quantile ranges from 1,000 simulations of resampled data from the selected simulation universe. The dotted “theoretical max” series presents the averages and ranges of Sharpe ratios for in-sample inequality unconstrained MV optimized MSR portfolios. For this case the exact simulation parameters with no estimation error are used. Of course the Sharpe ratios in this result series are unattainable in practice since they use unavailable inputs. However, the other three graphed series show ranges of Sharpe ratios resulting from estimates based on available data, simulated with realistic and perhaps optimistic error. The “unconstrained” series displays the out-of-sample averages and ranges of Sharpe ratios of the simulated unconstrained MSR portfolios. The “equal weight” series displays the average Sharpe ratios of equal weighted portfolios. The “constrained” series reflects the average Sharpe ratios of out-of-sample simulated long-only MSR portfolios. Intervals are also shown in all cases, showing the central 90% of the simulated information ratios. In other words, the crossed lines mark the 5% and 95% quantiles of the simulated portfolio Sharpe ratios. The intervals as shown on the page are jittered horizontally so as not to overlap and to maintain readability of the chart.

The simulations demonstrate the interaction of constraints, optimization methods, sample size, and information coefficients. The ”theoretical max” series assumes no estimation error in the optimization process. In this case the results are plausibly consistent with the GK view that adding assets increases the investment value of MV optimized portfolios. The unattainability of this level of performance in practice is clearly demonstrated by the inferior performance of the feasible methods.

Note that our simulations assume that the average level of IC is constant independent of universe size, ignoring any realistic limitations on manager information. Consequently, a larger universe corresponds to a larger level of investment information, all other things the same. As a result, the slowly rising level of unconstrained average maximum Sharpe ratios as universe size increases is a necessary artifact of the simulation framework. In practice, adding assets is unlikely to add investment value beyond some optimal size universe consistent with the investor's level of information all other things the same. Indeed, beyond some optimal point, the unconstrained curve is likely to curve downward as the size of the optimization universe increases in applications. Our experiment is designed with deliberate optimism to distill the impact of estimation error on the fundamental law's predicted performance.

The results of our experiments for the asset allocation cases demonstrate a definite failure of the GK and CST specifications of the fundamental law of management. In Figure 1A, the unconstrained portfolios dramatically underperform both sign constrained and equal weighting. While adding assets increases the Sharpe ratios of unconstrained portfolios out-of-sample, the gain is minimal and, we will argue below, unrealistic. How positivity constraints help the optimization process depends on the quality of information and universe size but the results generally contradict the CST view that eliminating constraints adds investment value.[30] In all cases in Figure 1, positivity constraints narrow the confidence intervals. The naïve analyst may think *a priori* that performance will increase because of increased IR forecasts calculated with estimates used in the optimization, but such in-sample calculation amounts to assuming perfect information and estimation ability, which is clearly unrealistic for investors of any skill level. Our results vividly demonstrate the hazards of ignoring estimation error when optimizing.

Our deliberate optimism in setting up the simulation has several important implications. The unconstrained cases would exhibit poorer performance with regard to IR in practice. Since almost all of the assets in the simulation universe are likely to have some investment value, the investor is not harmed by putting portfolio weight on the "wrong" assets. In the real world, in which the investable universe is not limited to stocks which will have any particular track records, constraints limit the harm caused by misinformation. This effect was clearly demonstrated and measured in Jobson and Korkie (1981). In a truly chaotic world with a lot of estimation error and bias, the equal weighted portfolio, which uses no information to distinguish among assets, can be hard to beat.

Figure 2 presents similar simulation experiments to Figure 1 for expanded optimization universes of up to 500 securities. One clear difference in the large universe case is the overall inferiority of equal weighting particularly given the presence of significant levels of information. A second important difference is that the benefit of positivity constraints depends crucially on the level of presumed forecast information. For a typical level of IC = 0.10, sign constrained large universe optimization provides similar performance relative to unconstrained for much of the size spectrum. In the likely unattainable level of IC = 0.50 associated with large stock universe equity portfolio optimizations as in Figure 2B, unconstrained dominates. These results should not be surprising; nor do they represent any

[30] We note that the results reaffirm the conclusions in Frost and Savarino (1988).

serious contradiction to our basic thesis that adding securities adds little if any investment value, all other things the same.

5.0 Prescriptions and Caveats

Merton (1987) posits that the appropriate optimization universe should be defined only for securities with reliable information. Intuitively, managers should optimize only on what they know. Investing in large stock universes that include many low-information securities is likely to be suboptimal. But the Merton rule is an important limitation in a number of practical applications. In addition, estimation error uncertainty remains a key consideration for effective asset management.

Figure 1A

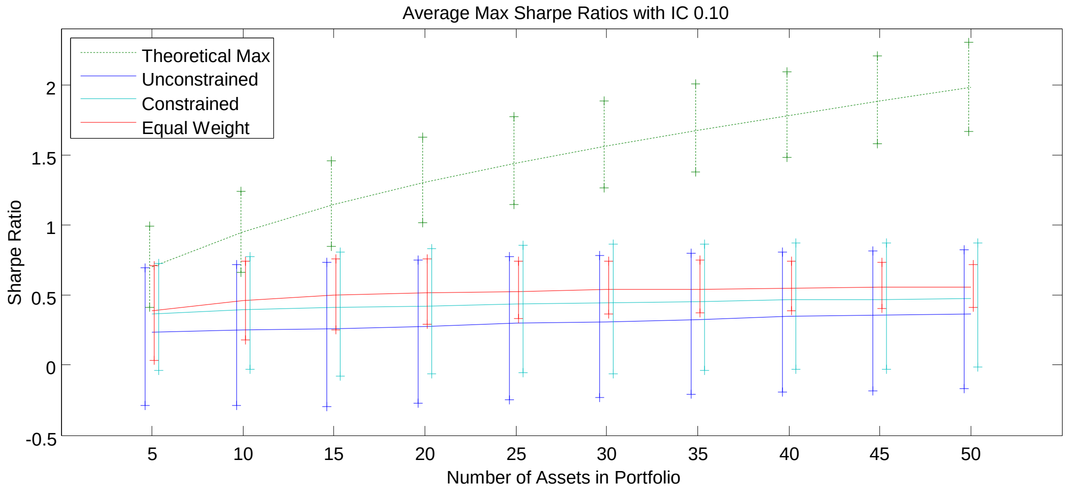

Figure 1B

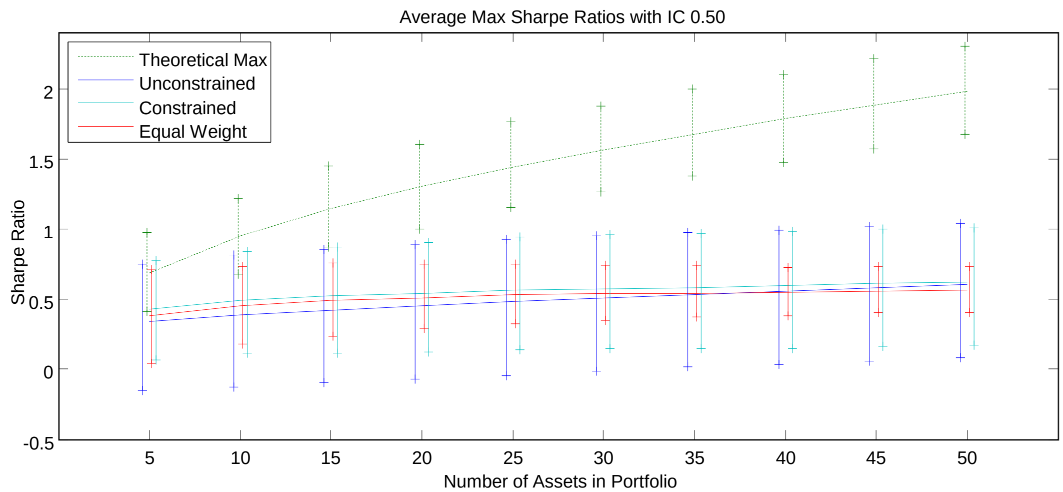

Figure 1: Maximum Sharpe Ratio ranges for three different portfolio construction methods and two different information coefficients for the asset allocation case, compared with corresponding ranges of MSRs of the unattainable perfect information frontiers. This experiment was run on many simulated resamplings of up to fifty U. S. stocks which had at least 2 years of contiguous monthly price data ending in December 2013.

5.1 The Composite Asset

Asset managers are often mandated to outperform an index while holding tracking error within a specified range. Defining the optimization universe solely for investment significant alpha securities may expose the optimized portfolio to unacceptable tracking error risk in the context of large stock universe benchmarks. Michaud and Michaud (2005) provide a reconciliation of these competing objectives. They recommend adding an index weighted "composite asset" to represent the non-investment significant alpha securities to the set of investment significant alpha securities as part of the optimization universe. Adding the composite asset does not violate Merton's theoretical prescription while satisfying the need for controlling tracking error risk in applications. Michaud and Michaud find that an optimized portfolio that includes consideration of estimation error with a composite asset for non-investment significant assets often exhibits very desirable optimization and portfolio risk characteristics.

Figure 2A

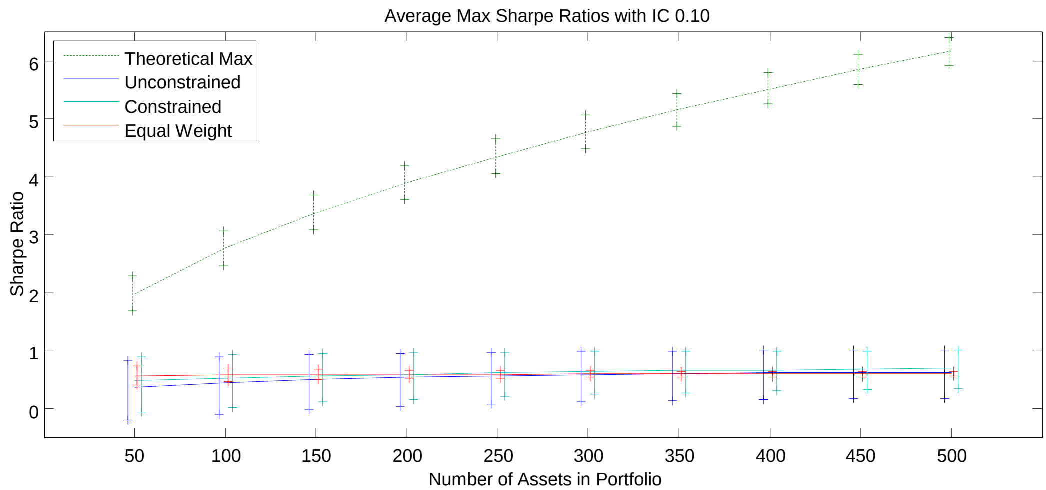

Figure 2B

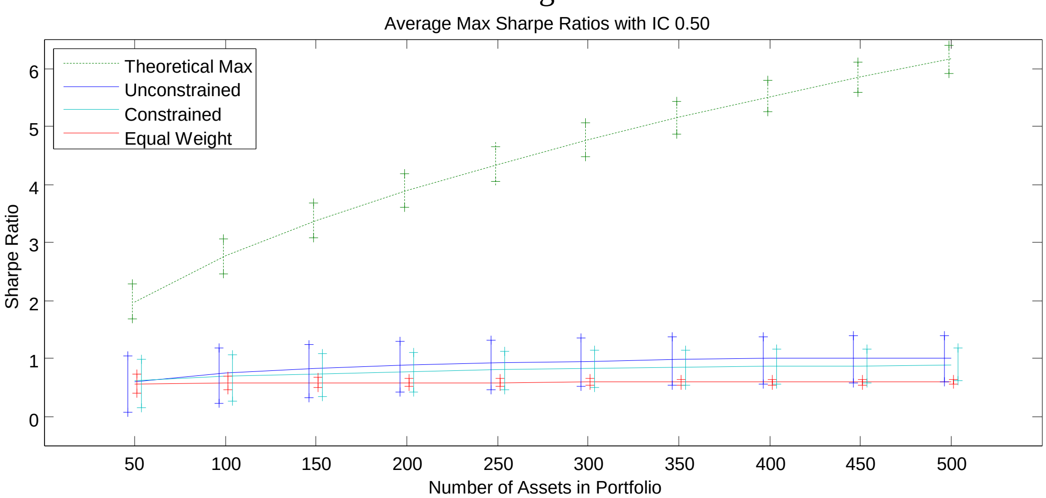

Figure 2: Maximum Sharpe Ratio ranges for three different portfolio construction methods and two different information coefficients for the equity portfolio case, compared with corresponding MSR ranges of the unattainable perfect information frontiers. This experiment was run on many resampled portfolios of up to five hundred U. S. stocks which had at least 2 years of contiguous monthly price data ending with December 2013.Figures 2A and 2B can be viewed as an extension of charts 1A and 1B to larger optimization universes.

5.2 Optimization in Uncertainty

Michaud (1998) and Michaud and Michaud (2008a, 2008b) MV efficient frontier optimization is a generalization of the linear constrained Markowitz efficient frontier that includes estimation error in investment information in its portfolio construction methodology.[31] Monte Carlo sampling of risk-return estimates from their uncertainty distributions is used to address uncertainty in investment information by creating thousands of statistically similar Markowitz MV efficient frontiers. An averaging process over these many alternatives defines the new Michaud efficient frontier.[32,33] Simulation studies demonstrate that the resulting efficient frontier portfolios have superior investment value on average out-of-sample relative to Markowitz.[34] In this case neither GK nor CST provide added investment value all things the same. The investment game mandates investment significant investment information thoughtfully considered and properly implemented.

5.3 Resolutions

Merton advocates investing in only what you know. Michaud advocates understanding how much you know, then modeling the remaining uncertainty in information when investing. Important considerations are often ignored by naïve interpretations of the fundamental law. GK is best understood as a theoretical framework for understanding the potential for performance but not a prescription for practice. Investment significant information may often be available for only a relatively small number of stocks in an index or optimization universe. Many managers intuitively understand that they should be investing in a set of assets for which they are best informed. Careful consideration of this issue can have dramatic implications for performance while avoiding investment ineffective strategies.

6.0 Summary and Conclusions

Making a number of assumptions, Grinold shows that the in-sample value added of MV inequality unconstrained optimized active investment strategies is approximately the product of the skill (IC) and square root of the breadth (BR) of the strategy. Based on the formula Grinold and Kahn argue that even a minimal amount of information can be used to enhance optimized portfolio investment value by trading more often and adding more

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[31] Michaud resampled optimization was invented by Richard Michaud and Robert Michaud and is a U.S patented procedure, #6,003,018; worldwide patents pending. It was originally described in Michaud (1998, Ch. 6). New Frontier Advisors, LLC (NFA) is exclusive worldwide licensee.

[32] Uncertainty level can be defined by a "forecast confidence" scale based on estimation periods as described in Michaud and Michaud (2008a, 2008b) or by the investor's IC.

[33] We note that Michaud optimization is not the same as the Morningstar resampling optimizer which uses a different frontier averaging process with different out-of-sample investment properties. See Michaud and Esch (2010) for further information.

[34] Michaud (1998, Ch. 6), Michaud and Michaud 2008a,b. Markowitz and Usmen (2003) simulation studies indicate that Michaud optimization may be superior to Markowitz even with inferior risk-return estimates.

assets and forecast factors. Clarke et al generalize the formula and recommend removing constraints in order to add investment value.

While the essential wisdom of the Grinold formula as a theoretical though unattainable upper bound is not controversial, we show that the GK and CST principles for active equity management derived from the formula are invalid and often self-defeating. This is because the Grinold formula ignores estimation error and the necessity of economically meaningful linear constraints for properly defining portfolio optimality. The fundamental limitations of the GK and CST principles are widespread in contemporary quantitative asset management and afflict many portfolio strategies in practice.

There are fundamental issues in contemporary science similar to the investment practice limitations chronicled in this report. Modern finance has inherited "a serious disconnect between quantitative research methodology and clinical practice."[35] Investment theory and applications are often based on in-sample (mathematical) frameworks that assume known probability distributions and stable returns similar to a casino game. But investing is not a casino game. Investors always have some level of uncertainty of their risk-return estimates in practice.[36] Simple prescriptions are often artifacts of theoretical models that do not consider the pervasive implications of estimation error on out-of-sample investment performance.[37] Necessary conditions for reliably winning the investment game include: 1) investment significant information; 2) economically meaningful constraints; and 3) properly implemented estimation error sensitive portfolio optimization technology.

[35] Weisberg op. cit., p. xiii.
[36] Assuming no illegal insider information.
[37] Weisberg (op. cit. p. xii) notes the need to reengineer probability by accounting for some of the complexity that has often been ignored.

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